



February 9, 2007

## Pair-wise cointegration in long-run growth models

This paper presents a novel theoretical and empirical approach to the analysis of long-run economic growth. It shows that most theoretical models share the feature of pair-wise cointegration among the main variables: the Solow model, the Lucas model and evolutionary models. An augmented Kaldor model is proposed in contrast to the standard production function.

The empirical analysis applies non-stationary panel techniques to two groups of countries to show that pair-wise cointegration exists among GDP, physical capital, human capital and trade openness. This is support for Nelson's (1981) view that "it may be fruitful to consider the several sources of growth as being like the inputs into a cake. All are needed." By contrast, the predictions of endogenous growth models with scale effects are rejected.

The theoretical and empirical approach of modeling long-run growth outlined here differs significantly from earlier studies: It neither attempts to estimate coefficients of an aggregate production function nor does it use investment ratios or population growth rates as explanatory variables.

The approach can be used for policy analysis. Policy priorities can be derived by identifying the weakest element among the three drivers of growth. For example, a country's physical capital and openness may currently be well above levels indicated by the cointegration relationships between these two variables and GDP, but human capital may be well below the equilibrium identified by the model. In this case, it would make little sense for that country to focus on even more accumulation of physical capital.

The setup can also be used for forecasting medium- to long-term GDP growth. This requires forecasts for human capital, openness and population. Forecast failure has been widespread in recent years. The information included in the trajectories of the growth drivers and in the deviations from the equilibria (cointegration errors) may help to reduce the size of these forecast errors.

Stefan Bergheim (stefan.bergheim@db.com)

www.  
dbresearch.com

**Editor**

Thorsten Nestmann

**Advisory Committee**

Dr. Peter Cornelius  
AlpInvest Partners

Prof. Soumitra Dutta  
INSEAD

Prof. Michael Frenkel  
WHU Koblenz

Prof. Helmut Reisen  
OECD Development Centre

Prof. Norbert Walter  
Deutsche Bank Research

Deutsche Bank Research  
Frankfurt am Main  
Germany

**Internet:** www.dbresearch.com

**E-mail:** marketing.dbr@db.com

**Fax:** +49 69 910-31877

**Managing Director**

Norbert Walter

# Pair-wise cointegration in long-run growth models

Stefan Bergheim<sup>1</sup>

This paper presents a novel theoretical and empirical approach to the analysis of long-run economic growth. It shows that most traditional theoretical models share the feature of pair-wise cointegration among the main variables. An augmented Kaldor model is proposed in contrast to the standard production function. The empirical analysis employs non-stationary panel techniques on two groups of countries to show that pair-wise cointegration exists among GDP, physical capital, human capital, and trade openness.

## 1 Introduction

Since Adam Smith wrote about the "Wealth of Nations" in the late 18th century, the analysis of long-run economic growth remains one of the most important topics in economics. It forms the basis for much of the policy advice economists give today and it is one of the reasons for the public's interest in economics. However, despite the progress made over the past decades, dissatisfaction with theoretical and empirical growth models remains widespread both within the profession and outside of it.

Theoretical models either have to rely on manna-from-heaven growth or they offer an overwhelming number of variables which all seem to be linked to growth. Even more, some endogenous growth models lead to predications that may not be helpful for explaining real world phenomena. Policy advice has certainly improved in recent years compared to earlier recommendations to simply increase the investment rate or to reduce the birth rate. Knowledge and institutions are currently attracting most attention. However, this new focus has not prevented sizeable mistakes in forecasting economies' long-run growth potential. For example, at the beginning of the 1990s, Japan was seen as the model economy while the US was seen as mired in stagnation. Reality showed that the reverse was true. Similarly, few observers were able to predict the downturn in Asian economies in the late 1990s - and even fewer expected the divergent paths of the recoveries of the different economies in the following years.

The still most-widely used method in empirical analysis is cross-section regressions even so they cannot possibly capture the inherently dynamic properties of long-run growth. Panel models have made inroads over the past decade but

---

<sup>1</sup>WHU Otto Beisheim School of Management, Vallendar and Deutsche Bank Research, stefan.bergheim@db.com. I would like to thank Michael Frenkel, seminar participants at ETH Zürich and Deutsche Bundesbank, and the referees Thorsten Nestmann and Peter Cornelius for valuable comments.

they do not always take into account the non-stationarity of the underlying variables. However, this non-stationarity may contain important information that can be uncovered through cointegration analysis: if two non-stationary variables do not stray far from a linear combination of the two, then there is a long-run equilibrium or cointegration relation between the variables. For example, GDP may converge to the path traced by one or more fundamental drivers of growth.

This paper proposes a novel theoretical and empirical approach to long-run growth analysis, which may help reconcile some of the theoretical disputes and improve the forecasting performance of growth models. Section 2 reviews the standard growth theories and emphasizes the result of pair-wise cointegration among the key variables. Section 3 presents an augmented Kaldor model making use of some stylized facts. Section 4 describes the data used for GDP, physical capital, human capital and trade openness. Section 5 presents the results from non-stationary panel estimation confirming the theoretical conclusion of pair-wise cointegration. Section 6 compares the framework of this paper to other approaches. Section 7 concludes and suggests ways to apply the framework in policy analysis and forecasting.

## 2 Growth theories revisited

Useful growth theories have to satisfy important conditions laid out by two Nobel Prize winners: Robert Solow (2001, p. 383) thinks "of growth theory as the search for a dynamic model that could explain the evolution of an economy over time" or equivalently "the theory of the evolution of potential output" (ibid p. 286). And Robert Lucas (1988, 5) defines theory as "an explicit dynamic system, something that can be put on a computer and run."

### 2.1 Neoclassical models

The neoclassical Solow model (Solow [1956], [1957]) remains the workhorse for growth empirics today and the benchmark against which all other models are compared. Solow analyzed a closed economy with only one good  $Y$ , no government involvement, a constant returns to scale production function, a constant and exogenous savings rate  $s$ , two factors of production capital  $K$  and labor  $L$ , and a level of technology  $A$  (or total factor productivity TFP) given exogenously. The standard Cobb-Douglas production function with Harrod-neutral technological progress is:

$$Y_t = K_t^\alpha (A_t L_t)^{(1-\alpha)} \quad (1)$$

where the subscript  $t$  denotes time and  $\alpha$  the elasticity of output with respect to capital input. Technology advances at the constant and exogenous rate  $g$ ,  $A_t = A_0 e^{gt}$ , and labor input at the constant and exogenous rate  $n$ ,  $L_t = L_0 e^{nt}$ .

Reformulating and expressing the key variables in per capita terms leads to the familiar conclusion: the growth rates of GDP per capita  $\hat{y}_t$  and of the capital stock per capita  $\hat{k}_t$  in the steady state are independent of the savings

rate but depend only on the pace of technological progress  $g$ .

$$\hat{y}_t = \hat{k}_t = \hat{A}_t = g \quad (2)$$

In econometric terminology, the three variables  $y$ ,  $k$ , and  $A$  are all driven by a single shock  $g$ . So there should be a cointegrating relationship among the two variables  $y$  and  $k$ .

The augmented Solow model made popular by Mankiw, Romer and Weil (1992) includes human capital as a third factor of production in the production function and leads to the same conclusion. All per capita variables grow at the same exogenous rate of  $g$  in the steady state:

$$\hat{y}_t = \hat{k}_t = \hat{h}_t = \hat{A}_t = g \quad (3)$$

Among the three integrated variables  $y$ ,  $k$ , and  $h$ , there should be two cointegrating relationships. Each pair of these three variables should be cointegrated.

A similar conclusion can be derived from the Lucas (1988) model without spillovers. Lucas introduced a second sector into the model economy, which produces human capital using a constant returns technology. This allowed him to express the long-run growth rate of GDP using parameters from within the model instead of having to rely on an exogenous rate of  $g$ . The relevant conclusion for the empirical analysis is that - just as in the (augmented) Solow model - all per capita variables grow at the same rate:

$$\hat{y}_t = \hat{k}_t = \hat{h}_t = \frac{1}{\sigma} (B - \delta - \rho) \quad (4)$$

This growth rate depends on the underlying characteristics of the economy: the rate of risk aversion  $\sigma$ , the technology of teaching  $B$ , the depreciation rate of (human) capital  $\delta$ , and the rate of time preference  $\rho$ . Again, a testable hypothesis is that  $y_t$ ,  $k_t$ , and  $h_t$  are pair-wise cointegrated.

Over the past decade growth theory increasingly focused on policies and institutions. Parente and Prescott (e. g. 1994, 2000, and 2005) propose to model the barriers preventing a country from using the best globally available technology as a way to explain differences in income levels as well as some "growth miracles" in the 20th century. Their theory of relative efficiencies (or of differences in total factor productivity TFP) decomposes a country's technology into a pure knowledge component common to all countries and an efficiency component specific to each country measuring the degree to which this country makes use of the available knowledge. The efficiency component captures, for example, work rules that require firms to use a minimum number of workers per machine or per plant.

Importantly for the analysis in this paper, it is impossible to combine free trade and barriers to entry/technology at the same time: If rules prevent domestic firms from using the best available technology, then foreign firms would be able to capture the market, if they were allowed to enter. Opening an economy to free trade is only sensible, if barriers to technology are reduced at the same time or earlier. In principle, a country can reduce its domestic barriers without opening to free trade. But then it will not be able to make use of all

the technology that is incorporated in imports or conveyed in the exchange with foreigners. The conclusion is that low barriers to entry and free trade tend to go hand in hand. Therefore, the degree of trade openness - for which data are easily available - should be a good measure of the efficiency component and possibly of the overall quality of a country's institutions. The empirical analysis below will show that trade openness  $o$  cointegrates with the other non-stationary variables in the model,  $y$ ,  $k$  and  $h$ .

## 2.2 Evolutionary models

Evolutionary models use bounded rationality, heterogeneity of agents, economic selection, and disequilibrium notions to model how economies evolve, i. e. their continuous change from a lower or simpler state to a higher or more complex state. They effectively come to the same conclusion of pair-wise cointegration as the models outlined above. Nelson (1981, page 42 in reprint) argues "it may be fruitful to consider the several sources of growth as being like the inputs into a cake. All are needed." Finding pair-wise cointegration among the key variables in an empirical growth model can be seen as support for this claim. Nelson (ibid) concludes that "where complementarity is important, it makes little sense to try to divide up the credit for growth, treating the factors as if they were not complements." However, this close linkage between the individual factors of growth leads to the question of what ultimately determines their evolution. Nelson also refers to the "general features of the economic environment and of political and social institutions that support all three sources and the growth they promote" which points to a the same research agenda that neoclassical growth theory has turned to in recent years: what ultimately explains the exogenous rate of technological progress that shapes all the proximate drivers of economic growth.

## 2.3 Models with scale effects

Models with knowledge spillovers have received a wide following in recent years, but remain subject to some controversy. As Jones (1999) outlined, the endogenous models with spillovers lead to predictions of scale effects, even if some papers claim otherwise in the title: The growth rate of per capita GDP depends positively either on the size of the population (or of human capital, see Romer [1990], Aghion and Howitt [1992] and others) or on the growth rate of the population (see Jones [1995], Segerstrom [1998] and Aghion and Howitt [2005]). These predictions have been criticized for example by Parente (2001) who argues that "the prediction known as scale effect is not born out by the data." Likewise, Bottazzi and Peri (2006) see "a clear rejection of the existence of a strong scale effect."

My empirical approach can also be used to test the hypotheses derived from this strand of endogenous growth models. For example, if the growth rate of GDP really depends on the level of human capital then the two series should be integrated of the same order. However, as will be shown below, the growth rate of GDP is stationary, while human capital is not stationary. Therefore

GDP growth and human capital cannot be cointegrated, which could be seen as evidence against models with scale effects. Brunner (2003) uses the same strategy in his investigation of the link between openness and GDP growth, but does not employ the powerful panel techniques used here.

### 3 The augmented Kaldor model

The neoclassical models presented above use an aggregate production function. However, aggregate production functions are a problematic concept and their coefficients cannot be estimated.<sup>2</sup> The good fit to the data is the main reason why aggregate production functions continue to be estimated. However, the high  $R^2$  stems from the fact that effectively the national income identity is estimated, where the value of output always has to equal the value of inputs, i. e. the wage sum plus the income from capital. The constancy of the factor shares in income over time and their similarity across countries ensures a good fit as Felipe and McCombie (2005) point out.

In light of these considerations, trying to estimate the coefficients of an aggregate production function is not a primary concern of this paper. An alternative avenue is advocated by Solow (2001), who suggests "to begin with unprejudiced empirical study of the speed of technological innovation." This is the approach to be followed below.

Nicholas Kaldor (1957) argued that the "purpose of a theory of economic growth is to show the nature of the non-economic variables which ultimately determine the rate at which the general level of production of an economy is growing" (p. 591). In his model, economic growth depends on the readiness of an economy to absorb technical change combined with the willingness to invest in physical capital (p. 599). He established the "stylized Kaldor facts" of a constant labor share in income, a constant capital-output ratio, and a constant rate of profit (i. e. the real interest rate) over time. Kaldor thought it is impossible to distinguish between a rise in output induced by new technology and one induced by additional capital because a higher capital stock per worker must inevitably have been preceded by the introduction of a superior technology.

Long-run GDP growth is exogenous in Kaldor's original model. His original "Technical Progress Function" using per-capita variables is:

$$\hat{y}_t = c + a\hat{k}_t \quad (5)$$

where  $c$  is a constant and  $a$  is a factor of proportionality. This technical progress function can be augmented to include human capital  $h$  and a variable that measures openness  $o$  capturing the efficiency component in the Parente/Prescott model outlined above.

An augmented technical progress function therefore is:

$$\hat{y}_t = a\hat{k}_t + b\dot{h}_t + (1 - a - b)\dot{o}_t \quad (6)$$

---

<sup>2</sup>The lack of a sound theoretical foundation was shown by Fisher (1969) and retraced in Cohen and Harcourt (2003). Felipe and Fisher (2003) show that aggregation of firm production functions is only valid under several extremely restrictive conditions.

Output grows at the weighted average rate of the percentage change of physical capital and the absolute changes of human capital and trade openness.<sup>3</sup> The time path of openness is assumed to be exogenous and  $\dot{h}_t$  may either be exogenous or it may be determined as in the simple Lucas model in a human capital producing sector. There is no constant  $c$  in the progress function anymore, assuming that human capital and openness comprehensively model technical progress.

A similar equation is shown in Bernanke and Gürkaynak (2001) and in Bottazzi and Peri (2007), however without any of the labels used in the present paper. The version in Bottazzi and Peri (2007) is  $\hat{y}_t = a\hat{k}_t + b\hat{A}_t$ , where  $A_t$  is the stock of total available scientific and technological ideas.

### 3.1 Pair-wise cointegration in the Kaldor model

Having established the augmented technical progress function, one can now invoke the stylized Kaldor-fact that physical capital and output grow at the same rate in the long run ( $\hat{k} = \hat{y}$ ) to ensure a constant capital-output ratio. Similarly, the absolute change of human capital is equal to a constant  $\omega$  times the growth rate of GDP per capita, as I will show in the empirical part:  $\dot{h} = \omega\hat{y}$ . Input this into 6 to get

$$\hat{k}_t = a\hat{k}_t + b\omega\hat{k}_t + (1 - a - b)\dot{o}_t \quad (7)$$

or equivalently

$$\hat{y}_t = \hat{k}_t = \omega\dot{h}_t = \lambda\dot{o}_t \quad (8)$$

where  $\lambda = \frac{(1-a-b)}{(1-a-\omega b)}$ . This close link between the growth rates of the main variables mirrors the results from the models summarized in equations 2 through 4. The testable hypothesis is the same: pair-wise cointegration among  $y_t$ ,  $k_t$ ,  $h_t$ , and  $o_t$ , although the slope coefficients on human capital (and trade openness) may differ from unity.

### 3.2 Capital formation and profitability

Physical capital plays a passive role in the Kaldor model. It adjusts to the intensity of innovation - driven in this augmented version by openness and human capital - which influences the profitability of new investment. This reasoning is in line with Prescott (1997) arguing that "the reason that capital per worker is high in rich countries is that total factor productivities are high in rich countries." A stronger development of openness and human capital raises the productivity of physical capital (the profit rate) and therefore the rate of capital accumulation. The capital stock per capita obeys the following rule:

$$k_t = \theta y_{t-1} + \phi \frac{\pi_{t-1}}{k_{t-1}} y_{t-1} \quad (9)$$

---

<sup>3</sup>Dots above a variable denote absolute changes. Labor economics also uses the absolute change rather than the percentage change of human capital to explain differences in income. My empirical analysis indicates that this is the appropriate approach for macroeconomic models as well.

It is equal to a coefficient  $\theta$  times output in the previous period plus a coefficient  $\phi$  times profits  $\pi$  in relation to the capital stock in the previous period. Reformulation of equation 9 leads to the rule for investment per capita  $i$  which depends in particular on the change in GDP and the change in the profit ratio:<sup>4</sup>

$$i_t = k_{t+1} - k_t = (y_t - y_{t-1})\left(\theta + \phi \frac{\pi_{t-1}}{k_{t-1}}\right) + \phi \left(\frac{\pi_t}{k_t} - \frac{\pi_{t-1}}{k_{t-1}}\right)y_t \quad (10)$$

This setup helps reconcile two views in the growth literature which are sometimes interpreted as opposing. One view argues that factor accumulation is not the main driver of growth, while the other view sees a strong positive link between investment and GDP growth. In the augmented Kaldor framework both views hold.

## 4 Data

The theoretical and empirical models in this paper are rather limited in size. They only include four variables: real GDP per capita, real physical capital per capita, human capital per capita and an adjusted trade share. The time series used for 40 countries run from 1971 to 2003 and stem from a variety of sources. The 40 countries are divided into a sub-sample of 21 rich countries and a second sub-sample of 19 emerging markets to account for the heterogeneity of the two groups.<sup>5</sup>

Real GDP per capita is taken from the World Bank's World Development Indicators. Data for the year 2000 are in constant 2000 international Dollars, while data for earlier and later years are calculated using the growth rates of GDP in local currency units. This procedure maintains the relative prices of the base year 2000 and, crucially, proceeds with the same growth rates as those published by national and international organizations. Per capita levels are calculated using population data from the Groningen Growth and Development Centre.

The stock of physical capital is calculated using the perpetual inventory method, similar to Hall and Jones (1999):  $K_t = K_{t-1} + I_t - \delta K_{t-1}$ , where  $\delta$  is the depreciation rate. Real investment is in local currency units and comes from the World Development Indicators. The capital stock in the base year is set equal to real gross investment in the base year divided by the sum of the average growth rate of investment and the depreciation rate. So  $K_{1971} = I_{1971}/(\kappa + \delta)$ . For the growth rate of investment  $\kappa$ , I use the grand mean of 2.4 percent for the rich countries and 5 percent for the emerging markets. The depreciation rate

---

<sup>4</sup>Equilibrium in the Kaldor model is ensured by assuming that the savings rate out of profit income exceeds the savings rate out of labor income by an amount larger than the sensitivity of the capital stock to the profit rate multiplied by the output-capital ratio i. e.  $s_\pi - s_L > \phi(Y/K)$ , see Kaldor (1957, 607).

<sup>5</sup>The countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK, USA, and Argentina, Brazil, Chile, China, Colombia, Egypt, India, Indonesia, Israel, Korea, Malaysia, Mexico, Nigeria, Philippines, Singapore, South Africa, Taiwan, Thailand, Turkey. They accounted for 86% of world GDP in PPP terms in 2005.

$\delta$  is set at 6 percent for all countries in all years, the same rate as in Hall and Jones (1999).<sup>6</sup> Data are converted to 2000 international Dollars using the same exchange rates as those implied in the GDP series. The data do not distinguish whether the capital stock is owned by residents or foreigners.

Human capital is measured by the average years of education of the population aged 25 to 64. Time series for 21 rich countries are based on a dataset from 1971 to 1998 published in Bassanini and Scarpetta (2001). They built on the work of De la Fuente and Doménech (2000), who in turn used the Barro-Lee data as a starting point. De la Fuente and Doménech (2000) and Portela et al. (2005) emphasize the measurement error in the Barro-Lee data that stems from the calculation of non-census observations using enrollment data. For the present paper the Bassanini and Scarpetta (2001) dataset is extended through 2003 using the OECD's Education at a Glance of 2005. For 19 emerging markets I use the dataset from Cohen and Soto (2001) with observations every 10 years between 1960 and 2010. Intermediate data were interpolated by spline. For Taiwan, the Barro-Lee data were used, extrapolated to 2003.

Finally, the openness measure is based on the share of nominal exports and imports in nominal GDP. Nominal ratios are preferable to real ratios since the relative price of traded and non-traded goods tends to change over time but that does not indicate a change of actual openness. Real trade shares rose much faster than nominal shares in the past as prices of traded goods rose less than prices of non-traded goods. Since the openness variable tries to capture the allocation of resources inside each individual country in each year it is necessary to use the relative prices between traded and non-traded goods prevailing in that year. When making cross-country comparisons of openness levels it is necessary to adjust the trade shares for country size because small countries tend to trade more across borders than large countries. No adjustment is made here for differences in price levels across countries as suggested by Alcalá and Ciccone (2004) because this would imply that the same 10 percentage point change of the nominal trade share would be seen as having a different impact on GDP depending on the price level.

## 5 The 2-stage estimation procedure

The estimation approach proposed here proceeds in two stages. The first stage uses non-stationary panel techniques to explore the long-run linkages between GDP and its drivers. The second stage is a stationary panel analysis which uses the estimated cointegrating errors from the first step and first differences of the drivers to estimate models for GDP growth. So far, methods to jointly estimate these two stages are not available for a system with several cointegrating relationships.<sup>7</sup> Each stage is applied separately to the two sub-samples.

---

<sup>6</sup>Other authors use slightly higher rates of 7% (Easterly and Levine [2001]) or lower rates of 5% (Prescott [1998] or Chen and Dahlman [2004]) or even 3% (Mankiw, Romer, and Weil [1992], Klenow and Rodriguez-Clare [1997] and Sarno [2001]).

<sup>7</sup>The likelihood-based test by Larsson et al. (2001) is so far not available for four or more variables, but would probably lead to results similar to those presented here.

The first stage starts with pre-testing for stationarity of the four series in the 40 countries. There is no consensus yet in the literature on panel unit root tests on when to use which estimator. Since Hlouskova and Wagner (2006) show the low power of tests where the null hypothesis is stationarity, only tests with the null hypothesis of a unit root are used here. Breitung (2000, UB in table 1) and Levin, Lin, and Chu (2002, LLC) use a homogeneous alternative, while Im, Pesaran, and Smith (2003, IPS) use a heterogeneous alternative.<sup>8</sup> As table 1 shows, all tests strongly agree that the logarithm of GDP per capita, the logarithm of physical capital per capita, the years of education and trade openness are non-stationary in the 21 rich countries. There is some disagreement about the non-stationarity of the openness series in the emerging markets group which stems from the fact that many emerging markets simply did not open up over the past decades. Tests using first differences reject the null hypotheses of unit roots throughout.

Table 1: Panel unit root tests

	21 rich countries				19 emerging markets			
	homogeneous		heterogeneous		homogeneous		heterogeneous	
	UB	LLC	IPS-t	IPS-LM	UB	LLC	IPS-t	IPS-LM
ln GDP p.c.	5.44 (1.00)	-0.19 (0.42)	7.34 (1.00)	-3.54 (1.00)	1.57 (0.94)	-0.02 (0.49)	0.46 (0.68)	0.33 (0.37)
ln Capital p.c.	2.82 (0.99)	3.04 (0.99)	7.27 (1.00)	-1.32 (0.91)	0.15 (0.56)	1.10 (0.87)	0.15 (0.56)	<b>3.71</b> <b>(0.00)</b>
Education	2.70 (0.99)	4.57 (1.00)	11.03 (1.00)	0.48 (0.32)	-1.11 (0.13)	1.13 (0.96)	1.04 (0.09)	<b>1.45</b> <b>(0.00)</b>
Openness	-0.54 (0.29)	3.02 (0.99)	3.24 (0.99)	-2.96 (0.99)	<b>-4.16</b> <b>(0.00)</b>	-0.12 (0.45)	<b>-5.12</b> <b>(0.00)</b>	<b>5.85</b> <b>(0.00)</b>

Sample length is 1971 to 2003. Lag length fixed at 3 for all tests, only intercept included. The LLC tests take adjustment terms for fixed time dimension T into account. P-values in parenthesis. Bold: reject H0 at the 5% level and conclude that series are stationary.

The results of these panel unit root tests can be interpreted as evidence against the models with scale effects sketched above. If the level of GDP and the level of human capital are integrated of the same order, then there cannot be a lasting impact of the level of human capital on the growth rate of GDP.

## 5.1 Pair-wise cointegration

The main contribution of this paper is to show that there are three cointegrating relationships among these four non-stationary variables in the two groups. A widely-used method for panel cointegration tests is the group mean (between) panel fully modified OLS (gFMOLS) method as outlined in Pedroni (2000). A homogeneous cointegration vector is estimated, but fixed effects and short-run dynamics are allowed to be unit-specific. The null hypothesis is for a common cointegrating vector, i. e. the variables cointegrate for each member of the panel with the same cointegrating vector  $\beta$ . The alternative hypothesis allows

<sup>8</sup>Estimation was done with the modified NPT made available by Jarka Hlouskova and Martin Wagner.

Table 2: Panel cointegration tests

left side	right side	21 rich countries		19 emerging markets	
		gFMOLS	2-step	gFMOLS	2-step
ln GDP p.c.	ln Capital p.c.	1.07 (76.3)	0.96 (29.2)	0.66 (47.1)	0.71 39.2
ln GDP p.c.	Education	0.23 (38.8)	0.20 (31.5)	0.03 (13.9)	0.23 (24.7)
ln GDP p.c.	Openness	0.06 (17.2)	0.04 (7.9)	0.003 (5.8)	0.02 (4.8)
ln Capital p.c.	Education	0.22 (40.3)	0.20 (23.0)	0.33 (29.3)	0.35 (27.2)
ln Capital p.c.	Openness	0.02 (15.0)	0.03 (9.8)	0.004 (5.7)	0.01 (3.0)
Education	Openness	0.12 (17.0)	0.10 (7.4)	0.01 (5.3)	0.02 (4.1)

Table shows coefficients from bi-variate panel cointegration tests. Null hypothesis is no cointegration. T-statistics in parenthesis.

unit-specific (heterogeneous) cointegration vectors. So the null hypothesis of no cointegration ( $\beta = 0$ ) is tested against the alternative of unit-specific cointegration. The estimator is based on the "between dimension" of the panel, i.e. on unit-by-unit estimation that allows different constants and slope coefficients. The mean value of the resulting slope coefficients can then be evaluated using a t-statistic constructed as the simple mean of the individual t-statistics.

Breitung (2005)<sup>9</sup> proposes a computationally convenient two-step estimator, where the short-run and the long-run parameters are estimated in two separate steps. In the first step, the unit-specific short-run parameters are estimated using separate models for all N cross sections allowing for unit-specific cointegration vectors. The second step estimates a common cointegration vector by OLS using a pooled regression. Breitung shows that the long-run parameters are asymptotically normally distributed. This second-step regression can be treated as ordinary least squares and the nonstationarity of the regressors can be ignored.

Table 2 shows the results from both tests for the two country groups. The logarithm of GDP per capita cointegrates with each of the three drivers and the three drivers cointegrate with each other with the expected coefficients. There is a pair-wise cointegration among the four variables as suggested by the theoretical models outlined above. The error terms from the three cointegrating relationships using GDP on the left-hand side are shown in the appendix for the 21 rich countries.

The cointegration between GDP and physical capital per capita is highly significant in both samples. For the rich countries the coefficient is close to unity, confirming the stylized fact established by Kaldor 50 years ago that the ratio between GDP and the capital stock is stationary. Exceptions from this

<sup>9</sup>The working paper version often quoted is Breitung (2002), the Gauss codes are available on his website.

common cointegrating vector are Ireland (likely because of rising net factor income on intellectual capital going abroad) and Japan (unusual investment incentives). The slope coefficient is smaller in the emerging markets, indicating a less efficient use of physical capital there. These results indicate that in the rich countries any 10% rise in the capital stock will go hand in hand with a 10% rise in GDP in the long run, partly because other drivers of growth will also increase. This interpretation differs from accounting-based exercises, which see a 10% rise in the capital stock leading to a 3.3% rise in GDP (capital's share in income being one third) ignoring the complementarity of the different drivers of growth. Figure 1 in the appendix illustrates that the cointegration errors are stationary, with the main exceptions in the rich countries being Ireland and Japan. There are no indications for non-linearities.

The cointegrating relationship between GDP per capita and the average years of education is also estimated with high statistical significance. The coefficients of 0.23 (gFMOLS) and 0.20 (2-step) in the 21 rich countries indicate that an additional year of education goes hand in hand with a roughly 20% rise in GDP per capita in the long run. Compared with the microeconomic returns of less than 10%, this value appears high at first sight. However, physical capital (and openness) also tends to rise with higher education. Therefore, the 20% return captures both direct and indirect effects of an increase in human capital.

Since one year of education is roughly a 10% increase in human capital in the 21 rich countries, these results do not support the prediction of the augmented Solow model that human capital and GDP grow at the same rate. The augmented Kaldor model is a more appropriate specification, with  $\omega$  in equation 8 equal to 0.2. In the emerging markets sample there also is a highly significant cointegrating relationship between GDP and human capital - with the coefficients slightly higher than in the rich countries. Figure 2 in the appendix shows large cointegration errors in some countries - of which some might stem from measurement error for human capital.

Not surprisingly, the cointegrating relationship between per capita GDP and trade openness produces lower t-statistics than for the other variables and there is some disagreement between the two tests about the size of the coefficient. However, the coefficient has the expected positive sign and is still estimated with a high level of statistical significance. In the rich countries the coefficients of 0.025 (gFMOLS) and 0.036 (2-step) indicate that a ten percentage point increase in the trade share goes hand in hand with a roughly 30% rise in GDP per capita in the long run. In the emerging markets sample, the estimated coefficients are much smaller with only the 2-step estimate of 0.018 close to the numbers from the rich countries.

So far, all cointegration tests had GDP on the left hand side. Other permutations confirm the theoretical insight that the variables are pair-wise cointegrated with transitivity holding. The relationship between physical capital on the left-hand side and human capital is estimated with high significance and the coefficients are almost exactly equal to those from the relationship between GDP and education. This makes sense given the coefficient of unity between GDP and physical capital. The same transitive result holds for the cointegration between physical capital and openness. The coefficients are similar in magni-

tude to those from the GDP-openness relationship. Here again, the coefficients are estimated with a relatively low precision. Finally, the cointegration relationship between education and openness is also significant with the coefficients roughly equal to the ratio of the coefficients from the individual relationships with GDP.<sup>10</sup>

## 5.2 The short-run panel models

The result of pair-wise cointegration cannot be the end of an investigation into the sources of economic growth. Cointegration is intimately connected to the idea of error-correction; the formal equivalence between cointegration and error correction has been established by Engle and Granger (1987). A deviation of a system from equilibrium conveys information regarding its likely future course and is therefore useful for forecasting. Furthermore, Maddala and Kim (1999, 189) point out that if two variables are cointegrated, at least one must Granger cause the other. In sum, a significant amount of information about economic growth may be derived from the cointegration relationships analyzed here.

Table 3: Models for per capita GDP growth 1971-2000

	Panel Rich	France	USA	Panel EM	Indonesia	Turkey
constant			1.52		-3.62	-3.03
CI yk (-1)	<b>-0.05</b>	-0.27	-0.19	<b>-0.05</b>		-0.13
CI yh (-1)	<b>-0.06</b>		-0.05			
CI yo (-1)		-0.04				-0.42
d capital	<b>0.85</b>	1.50		<b>0.60</b>	1.72	1.24
d education			9.28			
d open	<b>0.12</b>	0.19				
d hours	<b>0.46</b>	0.46	0.97			
d y (-1)				<b>0.10</b>		
adj. R2	<b>0.47</b>	0.60	0.83	<b>0.32</b>	0.54	0.60
Schwarz cr.	<b>4.13</b>	2.94	2.95	<b>5.65</b>	5.21	4.98

All coefficients significant at least at the 5% level. "CI yk" is the cointegration error between GDP and physical capital per capita.

The second stage of my empirical approach are short-run panel models for the growth rates of GDP in line with the augmented technical progress function in equation 6. The cointegrating errors capture the possibility that adjustment is not instantaneous so that economies are temporarily away from their steady state. In addition, the change in hours per capita is added to take care of possible changes in work effort that were not accounted for in equation 6. Table 3 shows examples of short-run models both for the two panels and for selected countries. In the rich countries, GDP adjusts to errors in the cointegrating relationships of GDP with physical and human capital. In the emerging markets sample, GDP corrects errors in the relationships with physical capital and trade openness.

<sup>10</sup>Given the pair-wise cointegration of the four variables it is not admissible to test whether, say, GDP cointegrates with both human capital and openness.

## 6 Relation to prior empirical work

The modeling approach in this paper differs from prior work in growth empirics both in the variables used and in the econometric techniques. The tested hypotheses are different as well. Most empirical growth analyses use production functions and the laws of motion of the exogenous variables to derive an empirical equation of the following form:

$$\ln y_i = a_0 + gt + \frac{\alpha}{1 - \alpha - \beta} \ln s_{k,i} + \frac{\beta}{1 - \alpha - \beta} \ln s_{h,i} - \frac{\alpha + \beta}{1 - \alpha - \beta} \ln (n_i + g + \delta) + \epsilon_i \quad (11)$$

where  $y$  is the level of per capita GDP,  $g$  is the growth rate of technological progress,  $t$  is time,  $s_k$  is saving for the accumulation of physical capital,  $s_h$  is saving for the accumulation of human capital,  $n$  is the growth rate of population,  $\delta$  is the depreciation rate, and  $\alpha$  and  $\beta$  are the elasticities of substitution for physical and human capital in the production function. The  $a_0$  is a measure of the initial level of technology, often approximated by additional variables. Subscripts  $i$  denote countries and an additional subscript  $t$  could denote the time dimension. Equation 11 is usually estimated using cross-section or panel techniques; sometimes it is interpreted as a cointegration relationship.

One difference to the approach proposed in this paper is that equation 11 uses the investment ratio instead of the level of physical capital to explain the level of income. This transformation requires that the capital-output ratios are identical across countries - an assumption that is usually not tested. It does not hold in the dataset used above, where the capital-output ratios over 1971-2003 range from less than 2 in the UK and in the US to around 3 in Germany and Finland.

The empirical finding of a positive coefficient on the investment ratio in cross-section studies stems in part from the use of international prices (Penn World Tables), which leads to extremely low investment ratios in low-income countries. In a sample of 19 empirical studies published between 1991 and 2004, 16 use investment ratios from the Penn World Tables and find a significant effect. Two of the three studies using IMF or OECD data find no effect and the final study seems to only capture an effect at the business cycle frequency.

Equation 11 also gives a prominent role to population growth which is assumed to be exogenous. However, demographic research clearly shows that population growth is not exogenous but depends on income. Richer societies tend to have fewer but better educated children. In the words of Becker, Glaeser and Murphy (1999, p. 146) "in the modern view, the growth in per capita income during the past 150 years has little to do with population." Likewise, Easterly (2001, p. 91) concludes "that there is no evidence one way or the other that population growth affects per capita growth."

Beyond focusing attention on a different set of variables, the approach presented in this paper also uses a different econometric approach. To date, cross-section methods remain prominent in growth empirics even if by construction they cannot model the evolution of economies over time. Solow (2001, p. 383) has "been skeptical from the beginning about the interpretation of cross-country

growth regressions.” The panel methods developed over the past decade helped researchers to model the dynamic properties of the growth process and the heterogeneity in the usually large sample of countries. Islam (1995) and Caselli, Esquivel and Lefort (1996) are important contributions. However, these methods do not explicitly deal with the non-stationarity of the time series. Pesaran, Shin and Smith (1999) propose a pooled mean group estimator that includes one cointegrating relationship in an error-correction model. However, cointegration is not tested and, as shown above, a single cointegrating relationship is too restrictive for the complex phenomenon of economic growth.

Few authors have approached long-run growth issues with cointegration techniques. One exception is Sarno (2001) who estimates country-specific cointegration vectors for the G7 economies for 1950 to 1992. The two main differences to my approach are that he looks for - and finds - just one cointegration relationship among five variables used in the augmented Solow model and that he does not use panel cointegration techniques. Dowrick (2004) also uses the idea of analyzing growth in an error-correction framework, but does not apply non-stationary panel techniques. Bottazzi and Peri (2007) use a similar approach as I do, with a stationary VAR as their second stage.

## 7 Conclusion and outlook

The theoretical and empirical approach of modeling long-run growth outlined in this paper differs significantly from earlier studies: It does not attempt to estimate coefficients of an aggregate production function and instead proposes an augmented Kaldor model with a technical progress function. The paper analyzes the linkages between four non-stationary variables with panel cointegration methods to find that all four variables considered here are closely linked to each other: pair-wise cointegration.

The theoretical model and the empirical results can be used for policy analysis. Policy priorities can be derived by identifying the weakest element among the three drivers of growth. For example, a country’s physical capital and openness may currently be well above levels indicated by the cointegration relationships between these two variables and GDP. But human capital may be well below the equilibrium identified by the model. Then it would make little sense for that country to focus on even more accumulation of physical capital. Instead, priority should be given to human capital. Furthermore, the fixed effects in the cointegration regressions may provide useful information about the general setup of the respective economies. For example, one country may consistently require a larger stock of human capital per unit of GDP than another country. In this case, further analysis could ask what accounts for this difference in average productivity and whether policy may want to act to address this difference. This will be the subject of future work.

The setup presented here can also be used for forecasting medium- to long-term GDP growth. This requires forecasts for human capital, openness and population, and is the subject of a companion paper. As highlighted above, forecast failure has been widespread in recent years. The information included in the trajectories of the growth drivers and in the deviations from the equilibria

(cointegration errors) may help reduce the size of these forecast errors.

Further work will try to improve and extend the model presented here. For example, data on physical capital from national statistical offices could be used instead of the crude data from the perpetual inventory method employed here. In addition, the stock of knowledge could be approximated using past expenditures on research and development. The model presented above could be used to test whether this stock of knowledge also cointegrates with the other determinants of GDP. Similarly, institutional quality may be a further variable that could be pair-wise cointegrated with the other variables in the model. However, long time series of meaningful measures of institutional quality are so far not available.<sup>11</sup> Furthermore, the short-run models could be extended by using determinants of short-run economic activity like interest rates or the oil price.

---

<sup>11</sup>The "Polity" database has long time series, but shows the maximum reading of 10 for most of the OECD countries throughout my 1971-2003 sample period.

## References

- Aghion, Philippe and Peter Howitt (1992): A model of growth through creative destruction. *Econometrica* 60(2), pp. 323-351.
- Aghion, Philippe and Peter Howitt (2005): Growth with quality-improving innovations: an integrated framework. In: Philippe Aghion and Steven Durlauf (eds.): *Handbook of Economic Growth*, Volume 1A, pp. 67-109.
- Alcalá, Francisco and Antonio Ciccone (2004): Trade and productivity. *Quarterly Journal of Economics* 119, pp. 612-645.
- Bassanini, Andrea and Stefano Scarpetta (2001): Does human capital matter for growth in OECD countries? *OECD Economics Working Papers* 282.
- Becker, Gary S., Edward L. Glaeser, and Kevin M. Murphy (1999): Population and economic growth. *American Economic Review* 89(2), Papers and Proceedings, pp. 145-149.
- Bernanke, Ben S. and Refet S. Gürkaynak (2001): Is growth exogenous? Taking Mankiw, Romer and Weil seriously. *NBER Macroeconomics Annual* 16, 11-57.
- Bottazzi, Laura and Giovanni Peri (2007): The dynamics of R&D and innovation in the short run and in the long run. *Economic Journal*, Forthcoming.
- Breitung, Jörg (2000): The local power of some unit root tests for panel data. In: Badi Baltagi (ed.): *Nonstationary Panels, Panel Cointegration, and Dynamic Panels*, *Advances in Econometrics*, Vol. 15, pp. 161-178.
- Breitung, Jörg (2005): A parametric approach to the estimation of cointegration vectors in panel data. *Econometric Reviews* 24, pp. 151-173.
- Brunner, Allan D. (2003): The long-run effects of trade on income and income growth. *IMF Working Paper No. 03/37*.
- Caselli, Francesco, Gerardo Esquivel, and Fernando Lefort (1996): Reopening the convergence debate: a new look at cross-country growth empirics. *Journal of Economic Growth* 1(3), pp. 363-390.
- Cohen, Avi J. and G.C. Harcourt (2003): Whatever happened to the Cambridge Capital Theory Controversies? *Journal of Economic Perspectives* 17(1), pp. 199-214.
- Cohen, Daniel und Marcelo Soto (2001): Growth and human capital: good data, good results. *OECD Development Centre Technical Paper* 179.
- De la Fuente, Angel und Rafael Doménech (2000): Human capital in growth regressions: how much difference does data quality make? *OECD Economics Working Paper* 262.

- Dowrick, Steve (2004): Ideas and education: level or growth effects and their implications for Australia. In: Ito, T. and Rose, AK. (eds): *Growth and Productivity in East Asia*, Chicago: Chicago University Press, pp. 9-37.
- Easterly, William (2001): *The elusive quest for growth: economists' adventures and misadventures in the tropics*. MIT Press.
- Engle, Robert F. and Clive W. J. Granger (1987): Cointegration and error-correction: Representation, estimation, and testing. *Econometrica* 55, pp. 251-276.
- Felipe, Jesus and Franklin M. Fisher (2003): Aggregation in production functions: What applied economists should know. *Metroeconomica* 54(2-3), pp. 208-262.
- Felipe, Jesus and John McCombie (2005): Why are some countries richer than others? A skeptical view of Mankiw-Romer-Weil's test of the neoclassical growth model. *Metroeconomica* 56(3), pp. 360-392.
- Fisher, Franklin M. (1969): The existence of aggregate production functions. *Econometrica* 37(4), pp. 553-577.
- Hall, Robert E. and Chad I. Jones (1999): Why do some countries produce so much more output per worker than others? *Quarterly Journal of Economics* 114, pp. 83-116.
- Hlouskova, Jaroslava and Martin Wagner (2006): The performance of panel unit root and stationary tests: Results from a large scale simulation exercise. *Econometric Reviews*, pp. 85-116.
- Im, Kyung So, M. Hashem Pesaran, and Yongcheol Shin (2003): Testing for unit roots in heterogeneous panels. *Journal of Econometrics* 115, pp. 53-74.
- Islam, Nazrul (1995): Growth empirics: A panel data approach. *Quarterly Journal of Economics* 110(4), pp. 1127-1170.
- Jones, Charles I. (1995): R&D-based models of economic growth. *Journal of Political Economy* 103(4), pp. 759-784.
- Jones, Charles I. (1999): Growth: with or without scale effects? *American Economic Review* 89(2), pp. 139-144.
- Kaldor, Nicholas (1957): A model of economic growth. *The Economic Journal*, pp. 591-624.
- Larsson, Rolf, Johan Lyhagen, and Michael Löthgren (2001): Likelihood based cointegration tests in heterogeneous panels. *Econometrics Journal* 4(1), pp. 109-142.
- Levin, Andrew, Chien-Fu Lin, and Chia-Shang Chu (2002): Unit root tests in

- panel data: asymptotic and finite-sample properties. *Journal of Econometrics* 108(1), pp. 1-24.
- Lucas, Robert (1988): On the mechanics of economic development. *Journal of Monetary Economics* 22, pp. 3 - 42.
- Maddala GS and In-Moo Kim (1999): Unit roots, cointegration and structural change. Cambridge University Press.
- Mankiw, Gregory N., David Romer, and David N. Weil (1992): A contribution to the empirics of economic growth. *Quarterly Journal of Economics*, pp. 407-437.
- Nelson, Richard R. (1981): Research on productivity growth and productivity differences: Dead ends and new departures, *Journal of Economic Literature* 19(3), pp. 1029-64. Reprinted in: Nelson, Richard R. (2005): *The sources of economic growth*. Harvard University Press, paperback edition, pp. 9-51.
- Parente, Stephen L. (2001): The failure of endogenous growth. *Knowledge, Technology, and Policy* 13(4), pp. 49-58.
- Parente, Stephen L. and Edward C. Prescott (1994): Barriers to technology adoption and development. *Journal of Political Economy* 102(2), pp. 298-321.
- Parente, Stephen L. and Edward C. Prescott (2000): *Barriers to riches*. MIT Press, Cambridge MA.
- Parente, Stephen L. and Edward C. Prescott (2005): A unified theory of the evolution of international income levels. In: Aghion, Philippe and Steven N. Durlauf: *The Handbook of Economic Growth*, Volume 1B, pp. 1371-1417.
- Pedroni, Peter (2000): Fully modified OLS for heterogeneous cointegrated panels, In: Badi H. Baltagi (ed.): *Advances in Econometrics*, Volume 15, Nonstationary Panels, Panel Cointegration, and Dynamic Panels, pp. 93-130.
- Pesaran, M. Hashem, Yongcheol Shin, and Ron P. Smith (1999): Pooled mean group estimation of dynamic heterogeneous panels. *Journal of the American Statistical Association* 94(446), pp. 621-634.
- Portela, Miguel, Rob Alessie and Coen Teulings (2005): Measurement error in education and growth regressions. Tinbergen Institute Discussion Paper 040/03. Version dated November 2005.
- Prescott, Edward C. (1997): Needed: A theory of total factor productivity. *International Economic Review* 39(3), pp. 525-551.
- Romer, Paul (1990): Endogenous technological change. *Journal of Political Economy* 98(5), pp. S71-S102.
- Sarno, Lucio (2001): Nonlinear dynamics, spillovers and growth in the G7 economies:

An empirical investigation. *Economica* 68, pp. 401-426.

Seegerstrom, Paul A. (1998): Endogenous growth without scale effects. *American Economic Review* 88(5), pp. 1290-1310.

Solow, Robert M. (1956): A contribution to the theory of economic growth. *Quarterly Journal of Economics* 70, pp. 65-94.

Solow, Robert M. (1957): Technical change and the aggregate production function. *The Review of Economics and Statistics* 39, pp. 312-320.

Solow, Robert M. (1987): Nobel Prize Lecture. (Includes an add-on from 2001).

Solow, Robert M. (2001): What have we learned from a decade of empirical research on growth? Applying growth theory across countries. *World Bank Economic Review* 15, pp. 283-288.

## Appendix

Figure 1: Errors from cointegration between GDP and capital stock

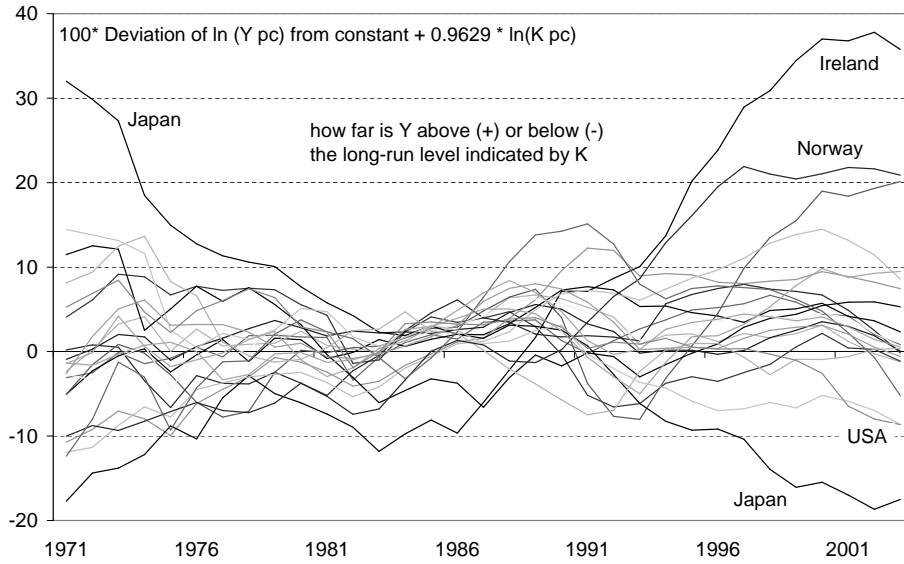


Figure 2: Errors from cointegration between GDP and years of education

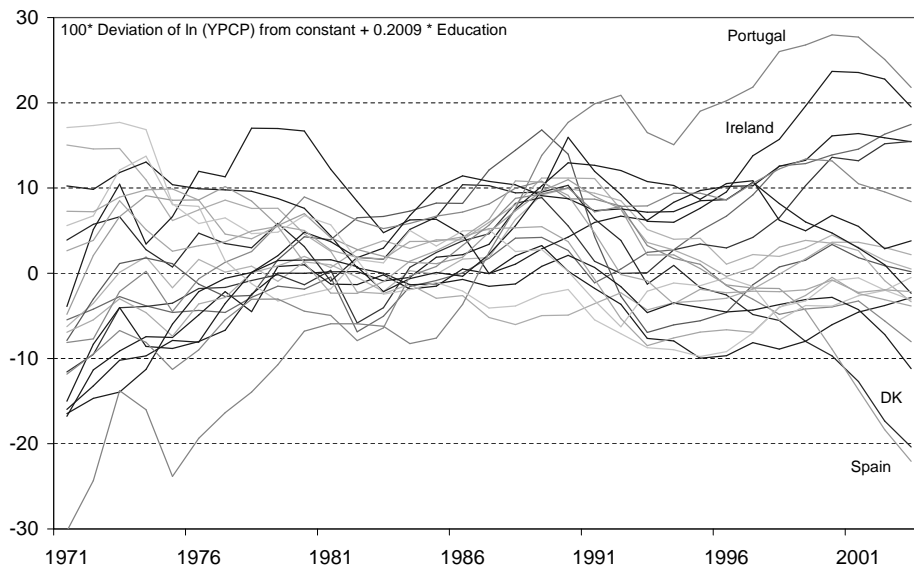
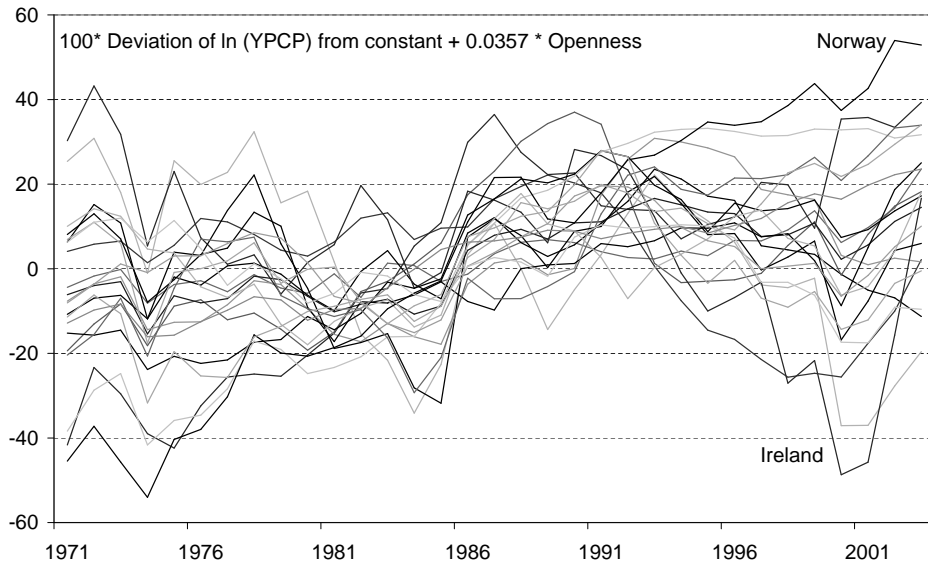


Figure 3: Errors from cointegration between GDP and trade openness



© Copyright 2007. Deutsche Bank AG, DB Research, D-60262 Frankfurt am Main, Germany. All rights reserved. When quoting please cite "Deutsche Bank Research".

The above information does not constitute the provision of investment, legal or tax advice. Any views expressed reflect the current views of the author, which do not necessarily correspond to the opinions of Deutsche Bank AG or its affiliates. Opinions expressed may change without notice. Opinions expressed may differ from views set out in other documents, including research, published by Deutsche Bank. The above information is provided for informational purposes only and without any obligation, whether contractual or otherwise. No warranty or representation is made as to the correctness, completeness and accuracy of the information given or the assessments made.

In Germany this information is approved and/or communicated by Deutsche Bank AG Frankfurt, authorised by Bundesanstalt für Finanzdienstleistungsaufsicht. In the United Kingdom this information is approved and/or communicated by Deutsche Bank AG London, a member of the London Stock Exchange regulated by the Financial Services Authority for the conduct of investment business in the UK. This information is distributed in Hong Kong by Deutsche Bank AG, Hong Kong Branch, in Korea by Deutsche Securities Korea Co. and in Singapore by Deutsche Bank AG, Singapore Branch. In Japan this information is approved and/or distributed by Deutsche Securities Limited, Tokyo Branch. In Australia, retail clients should obtain a copy of a Product Disclosure Statement (PDS) relating to any financial product referred to in this report and consider the PDS before making any decision about whether to acquire the product.

Printed by: HST Offsetdruck Schadt & Tetzlaff GbR, Dieburg